

CHAPTER – 2

LITERARY RESEARCH

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2.0 LITERARY RESEARCH ON PRĀṆĀYĀMA AND VEDIC MATHS

2.1 BACKGROUND AND SCOPE

Prāṇā and its flow is mentioned in several Upaniśads, Bhagavad Gītā, Ayurveda and Yogā texts. Different schools of Haṭh Yogā or Rāja Yogā or Kriyā Yogā give importance to Prāṇāyāma to explore a human being's hidden potential. The whole idea of practising Prāṇāyāma is to accelerate the spiritual journey. Can this component of Yogā be used to manage the cognitive development of a school going child? Can it be used to treat the issue of Math Anxiety? In this context different ancient scriptures are studied. The theoretical framework of the Prāṇā and Prāṇāyāma according to the yogic culture is established. Examples are taken from ancient classical books which specifically address the use of Prāṇāyāma and the method of practising it.

All Sūtras discussed in different modern Vedic Maths books are given with examples.

The origin of the Sūtras is traced in ancient texts.

2.2 AIMS AND OBJECTIVES

Aim: To understand the theory of Prāṇāyāma and Vedic Maths.

Objectives:

- To understand the concept of Prāṇā
- To establish the theoretical framework of Prāṇāyāma and its effects
- To find the origin of Vedic Maths

2.3 SOURCE MATERIAL

- Praśnopaniśad
- Taittirīya Upaniśad
- Suśruta saṁhita
- Kaṭha Upaniśad
- Chāndogyopaniśad
- Muṇḍaka Upaniśad
- Yogākuṇḍalinī Upaniśad
- Muktikopaniśad
- Bhagavad Gītā
- Śiva Saṁhitā
- Yogā Cuḍāmaṇi Upaniśad
- Amṛtānanda Upaniśad
- Patañjali Yogā Sūtras
- Aparokṣānubhuti
- Haṭh Yogā pradipika
- Bodhāyana Śulba Sūtra
- Āpastamba Śulba Sūtra
- Kātyātana Śulba Sūtra
- Vedic Maths books

2.4 CONCEPT OF PRĀṆĀ

2.4.1 Prāṇā:

Prāṇā is a Sanskrit word with two syllables, 'Prā', meaning constant, and 'ṇā' meaning movement. The word Prāṇā thus means *constant movement*. Its presence is life; life departs when Prāṇā goes. In this context, the Praśnopaniśad states that the entire universe is a manifestation of Prāṇā.

प्राणस्येदं वशे सर्वं त्रिदिवे यत् प्रतिष्ठितम् ।

मातेव पुत्रान् रक्षस्व श्रीश्च प्रज्ञां च विधेहि न इति ॥२॥१३॥

prāṇasyedaṁ vaśe sarvaṁ tridive yat pratiṣṭhitam |

māteva putrān rakṣasva śrīśca prajñāṁ ca vidhehi na iti ||2||13||

All that exists in the three worlds rests under the control of Prāṇā.

As a mother protects her children, O Prāṇā, protect us and give us splendor and wisdom.

Praśnopaniśad, Ch: 2 V: 13

2.4.2 Prāṇāmaya Kośa:

Taittirīya Upaniśad presents a broader picture of human existence using stories. At one stage of exploration, the future Bhṛgu Mahārṣi who was trying to understand and questioning layers of existence, concluded that Prāna is his identity. Later he realized that Prāna is only the second of five layers of his existence: First, the physical body; second, the pranic body; third, the mental body; fourth, the intellectual body; and fifth, the spiritual, causal body.

प्राणो ब्रह्मोति व्यजानात् । प्राणाद्धयेव् खल्विमानि भूतानि जायन्ते ।

प्राणेन जातानि जीवन्ति । प्राणं प्रयन्त्यभिसंविशन्तीति ॥३॥३॥१॥

prāṇo brahmoti vyajānāt | prāṇāddhayev khalvimāni bhūtāni jāyante |
prāṇena jātāni jīvanti | prāṇaṃ prayantyabhisamviśantīti ||3|3|1||

He identified prāṇa as Brahman.

Through Prāṇa all things live,

Through Prāṇa are sustained,

and into Prāṇa all things dissolve at the time of departure.

Taittirīya Upaniṣad, Ch: 3.3 V: 1

2.4.3 All-pervading Prāṇā:

अरा इव रथनाभौ प्राणे सर्वं प्रतिष्ठितम् ।

ऋचो यजूंषि सामानि यज्ञः क्षत्रं ब्रह्म च ॥२॥६॥

arā iva rathanābhau prāṇe sarvaṃ pratiṣṭhitam |

ṛco yajūṃṣi sāmāni yajñāḥ kṣatram brahma ca ||2|6||

Like spokes on the hub of a chariot wheel, all things are fixed on Prāṇa –

Ṛks, Yajus, Sāmas, sacrifice, Kṣatriya and Brahmaṇa.

Praśnopaniṣad, Ch: 2 V: 6

2.4.4 Gross manifestation of Prāṇā:

आत्मन एष प्राणो जायते ।

यथैषा पुरुषे छा्यैतस्मिन्नेतदाततं मनोकृतेनायात्यस्मिञ्शरीरे ॥३॥३॥

ātmana eṣa prāṇo jāyate |

yathaiṣā puruṣe chāyaitasminnetadātataṃ manokṛtenāyātyasmiñśarīre ||3|3||

*This Prāṇa comes from the Self. Just as a body has its shadow, similarly, Prāṇa is
inherent in the self, and when it wishes, it assumes a gross body.*

Praśnopaniṣad, Ch: 3 V: 3

2.4.5 Prāṇā the Lord: Prāṇa is considered as different deities.

इन्द्रस्त्वं प्राण तेजसा रुद्रोऽसि परिरक्षिता ।

त्वमन्तरिक्षे चरसि सूर्यस्त्वं ज्योतिषां पतिः ॥२॥९॥

indrastvaṃ prāṇa tejasā rudro'si parirakṣitā |
tvamantarikṣe carasi sūryastvaṃ jyotiṣāṃ patiḥ ||2|9||

O Prāṇa, you are Indra, you are Rudra; and you are the preserver on all sides.

You move in the sky – you are the Sun, the Lord of all luminaries.

Praśnopaniśad, Ch: 2 V: 9

प्रजापतिश्चरसि गर्भे त्वमेव प्रतिजायसे ।

तुभ्यं प्राण प्रजास्त्विमा बलिं हरन्ति यः प्राणैः प्रतितिष्ठसि ॥२१७॥

prajāpatiścarasi garbhe tvameva pratijāyase |
tubhyaṃ prāṇa prajāstvimā baliṃ haranti yaḥ prāṇaiḥ pratitiṣṭhasi ||2|7||

It is you who move about in the womb as the Lord of creation, and it is you who take birth after the image of the parents. O Prāṇā, it is you, who reside with the organs that all these creatures carry presents.

Praśnopaniśad, Ch: 2 V: 7

व्रात्यस्त्वं प्राणैर्कर्षिरत्ता विश्वस्य सत्पतिः ।

वयमाद्यस्य दातारः पिता त्वं मातरिश्च नः ॥२११॥

vrātyastvaṃ prāṇaikarṣirattā viśvasya satpatiḥ |
vayamādyasya dātāraḥ pitā tvaṃ mātariśva naḥ ||2|11||

O Prāṇa, you are beyond purifying process unpurified, you are the fire Ekarsi, (you are) the eater, and you are the lord of all that exists. We are the givers of (your) food. O Matarisva, you are our father.

Praśnopaniśad, Ch: 2 V: 11

एषोऽग्निस्तपत्येष सूर्य एष पर्जन्यो मघवानेव वायुः ।

एष पृथिवी रयिर्देवः सदसच्चामृतं च यत् ॥२१५॥

eṣo'gnistapatyeṣa sūrya eṣa parjanya maghavāneva vāyuḥ |
eṣa pṛthivī rayirdevaḥ sadasaccāmṛtaṃ ca yat ||2|5||

*This one (Prāṇa) burns as fire, it is the sun, cloud, Indra and air, the earth and food.
This God is both gross and the subtle, as well as that which is nectar.*

Praśnopaniṣad, Ch: 2 V: 5

2.4.6 Source of Prāṇa:

Food and water are sources of Prāṇa. Chāndogyopaniṣad explains how water becomes Prāṇa.

आपः पीतास्त्रेधा विधीयन्ते तासां यः स्थविष्ठो धातुस्तन्मूत्रं भवति यो मध्यस्तल्लोहितं योऽणिष्ठः स
प्राणः ॥५॥२॥

āpaḥ pītāstredhā vidhīyante tāsāṃ yaḥ sthaviṣṭho dhātustanmūtraṁ bhavati
yo madhyastallohitaṁ yo'ṇiṣṭhaḥ sa prāṇaḥ ॥5॥2॥

*Water, when drunk, becomes divided into three parts. Its grossest part, becomes urine;
the middle part becomes blood; and its subtlest aspect, becomes Prāṇa.*

Chāndogyopaniṣad, Ch: 5 V: 2

2.4.7 Prāṇa Sacrifice:

सर्वाणीन्द्रियकर्माणि प्राणकर्माणि चापरे ।

आत्मसंयमयोगाग्नौ जुहति ज्ञानदीपिते ॥४॥२७॥

sarvāṇīndriyakarmāṇi prāṇakarmāṇi cāpare ।

ātmasaṁyamayogāgnau juhvati jñānadīpīte ॥4॥27॥

Bhagavadgīta, Ch: 4 V:27.

*Others (pursuing self-realization through meditation), offer karmic fruits of all sense-
perception and the vital breath into the fire of knowledge kindled by Yogā.*

2.5 TYPES OF PRĀṆĀ

2.5.1 Pañca Prāṇā

Prāṇā is classified into five major categories and five subcategories (Upaprāṇa) by movement and direction.

अत्रापि वायवः पंच मुख्याः स्युर्दशतः पुनः ।

तत्रापि श्रेष्ठकर्त्तरौ प्राणापानौ मयोदितौ ॥३।६॥

atrāpi vāyavaḥ pañca mukhyāḥ syurdaśataḥ punaḥ |
tatrāpi śreṣṭhakarttārau prāṇāpānau mayoditau ||3|6||

*Out of the ten (major and minor Prāṇās), the first five are the leading ones;
among these, Prāṇā and Apāna are the highest agents.*

Śiva Samhitā, Ch: 3 V: 6

हृदि प्राणो गुदेऽपानः समानो नाभिमण्डले ।

उदानः कण्ठदेशस्थो व्यानः सर्वशरीरगः ॥३।७॥

hr̥di prāṇo gude'pānaḥ samāno nābhimaṇḍale |
udānaḥ kaṇṭhadeśastho vyānaḥ sarvaśarīragaḥ ||3|7||

*The seat of the Prāṇā is the heart; of the Apāna anus; of the Samāna the region about
the navel; of the Udāna the throat; while the Vyāna moves throughout the body.*

Śiva Samhitā, Ch: 3 V: 7

आक्षिप्तो भुजदण्डेन तथा यथोच्चलति कन्दुकः ।

प्राणापानसमाक्षिप्तस्तथा जीवो न तिष्ठति ॥२७॥

ākṣipto bhujadaṇḍena tathā yathoccalati kandukaḥ |
prāṇāpānasamākṣiptastathā jīvo na tiṣṭhati ||27||

*Just as a ball goes up and down when thrown by the hand, similarly, the individual
soul is thrown up (and down) by the movement of Prāṇā and Apāna (and therefore)
does not remain still.*

Yogā Cuḍāmaṇi Upaniśad, V 27

रक्तवर्णो मणिप्रख्यः प्राणवायु प्रकीर्तितः ।

अपानस्तस्य मध्ये तु इंद्रकोपसमप्रभः ।

समानस्तु द्वयोर्मध्ये गोक्षीरधवलप्रभः ।

आपाण्डर उदानश्च व्यानो ह्यर्चिसमप्रभः ॥३४-३७ ॥

raktavarṇo maṇiprakhyaḥ prāṇavāyu prakīrtitaḥ |
apānastasya madhye tu indrakopasamaprabhaḥ |
samānastu dvayormadhye gokṣīradhavalaprabhaḥ |
āpāṇḍara udānaśca vyāno hyarcisamaprabhaḥ ||34-37||

Prāṇā is said to be blood-red, the colour of ruby or coral; Apāna, the colour of indra – gopa (a white or red coloured insect); samāna is between the colour of pure milk and crystal (oily and shining); udāna is apandara (pale white); while vyāna is the colour of archis (a ray of light).

Amṛtānanda Upaniśad, V-34-37

Prāṇā: It flows in the Mouth Cavity and temporal region. Its Movement is either inward or upward. It governs eating, drinking, and inhalation, and all kinds of receptive action into body and mind.

Apāna: flows in the lower region of the intestines, normally downward, and in the urinary and reproductive systems. It is responsible for urination and defecation, ejaculation, menstruation, and giving birth; also, exhalation of carbon dioxide. Likewise, it eliminates negative emotional or mental experiences.

Udāna: It flows circularly in the throat region, governing speech.

Samāna: flows in the region of the stomach and naval governing peristalsis and nutrient absorption. Its movement is from periphery to the centre. It governs both digestion and cellular processes.

Vyāna: is active on the surface of the body. Its movement is from the center to the periphery. It is responsible for expansion and contraction processes of the body, and for lymph flow.

The Nāga, Kūrma, Kṛkara, Devdatta, Dhanañjaya Prāṇās are the five Upaprāṇa. The Praśnopaniśad says that these influence and alter physical existence.

The role of the Prāṇā is movement. When the body drops off the soul moves on, carried by the value of its deeds in this and its past lives. Traditionally the Yogi makes sure he blocks off the path of Apāna, which will carry him down to the patalas, so that the upward moving power of Udāna. Prāṇā can carry his jīva to the heavenly worlds of virtuous.

The cosmic correspondences are: the sun is the reservoir of cosmic light or Prāṇā. He rises helping the microcosmic Prāṇā to see objects i.e. he aids cognition. The goddess who presides over Earth (bhūmi) is the cosmic Apāna; she helps man by attracting the person's downwards-directed Apāna. The space between the Earth and the Sun (antarikṣa) is the cosmic Samāna, (which helps man's internal Samāna). The atmosphere, Vāyu, is the cosmic Vyāna (helping man's internal Vyāna)

एषोऽणुरात्मा चेतसा वेदितव्यो यस्मिन्प्राणः पञ्चधा संविवेश ।

प्राणौक्षित्तं सर्वमोतं प्रजानां यस्मिन्विशुद्धे विभवत्येष आत्मा ॥९॥

eṣo'ṅurātmā cetasā veditavyo yasminprāṇaḥ pañcadhā saṁviveśa |
prāṇaukṣittam sarvamotaṁ prajānāṁ yasminviśuddhe vibhavatyēṣa
ātmā ||9||

This subtle Ātman should be known by the mind as being in the body, whose Prāṇā entered in five different forms; the mind in all creatures is pervaded by these Prāṇās. When it is purified, then the Ātman shines out of itself.

Muṇḍaka Upaniṣad 3.1.9

The individual soul can be perceived by perfect intelligence as floating in five kinds of life force (Prāṇā, Apāna, Vyāna, Samāna and Udāna). When consciousness (pervading the soul and the entire body) is purified from contamination by these five, its spiritual nature is exhibited / shines forth.

This Ātman, who is thus seen, is subtle and should be known by the mind, *i.e.*, by the mere intellect purified. Where is this Ātman? In the body which, Prāṇā in five different forms, has well entered. The meaning is; he should be known by the mind as existing in the body, *i.e.*, in the heart; by mind, how circumstanced should he be known, is explained; mind in all creatures is pervaded by the Prāṇās and the senses, as milk by oil, and fuel by fire. The mind in all living beings is well known in the world, to be possessed of intelligence; when the mind is purified, *i.e.*, freed from the taint of grief, etc., then this Ātman above-defined shines out, shows itself out, by itself.

2.5.2 Disturbances in Prāṇā

ऋतं पिबन्तौ सुकृतस्य लोके गुहां प्रविष्टौ परमे परार्धे ।

छायातपौ ब्रह्मविदो वदन्ति पञ्चाग्नयो ये च त्रिणाचिकेताः ॥१३॥१॥

ṛtaṁ pibantau sukṛtasya loke guhāṁ praviṣṭau parame parārdhe |
chāyātapau brahmavido vadanti pañcāgnayo ye ca triṇāciketāḥ ||13||1||

*Both enjoyers of the fruits of their good deeds
are lodged in the cavity of the heart, the seat of the supreme.*

*Them knowers of Brahman call shadow and light, as also those who maintain
five fires and have thrice propitiated the Nachikētas fire.*

Kaṭha Upaniṣad 1.3.1

Naciketā and Yamā represent the Jiva and its transcendental basis, the supreme soul. Both reside in the cave of the heart. In that cavity, the living being, resting on the main Prāṇā, enjoys the results of activities, while the *supreme*, acting as witness enables him to enjoy them. Both those well-versed in knowledge of Brahman, and householders scrupulously following Vedic rules of life, state that the difference between the two is like the difference between a shadow and the sun.

The Yoga Vaśiṣṭha and Bhagavadgītā, both explain that desire leads to mental disturbances, ādhi, in the Manomaya Kośa. Such disturbances cause *pranic* imbalances in the Prāṇamaya Kośa, and percolate to the Annāmaya Kośa as vyādhi (disease) (Saini, 2011; Nagendra et al., 2010), The root cause of any anxiety or stress-related problems lies in mental and pranic disturbance.

दिवा सुप्तिर्निशायां तु जागरादतिमैथुनात् ।

बहुसंक्रमणं नित्यं रोधान्मूत्रपुरीषयो ॥

divā suptirniśāyāṃ tu jāgarādatimaithunāt |
bahusaṅkramaṇaṃ nityaṃ rodhānmūtrapuriṣayo ||

विषमाशन्दोषाच्च प्रयासप्राणचिन्तनात् ।

शीघ्रमुत्पद्यते रोगः स्तम्भयेद्यदि संयमी ॥१॥५६-५७ ॥

viṣamāśandoṣācca prayāsaprāṇacintanāt |
śīghramutpadyate rogaḥ stambhayedyadi saṃyamī ||1.56-1.57||

Physical diseases are generated by the following:

*sleeping in daytime, overnight vigils, excess sexual intercourse, moving in crowds,
withholding discharge of urine and faeces, the evil of unwholesome food,
and excessive use of Prāṇa.*

We should note that many present-day students reluctantly do laborious mathematics and science home-work to achieve in pressurized exams. Some may work overnight and sleep in daytime; the Massachusetts Institute of Technology library is open all night and students sleep on the benches!

भावसंवित्प्रकटितामनुरुपा च मारुते ।
चित्तस्योत्पत्युपरमा वासनां मुनयो विदुः ॥२॥२४॥
वासनावशतः प्राणस्पन्दस्तेन च वासना ।
क्रियते चित्तबीजस्य तेन बीजाङ्कुरक्रमः ॥२॥२६॥
bhāvasānvitprakaṭitāmanurupā ca mārute |
cittasyotpatyuparamā vāsanāṁ munayo viduḥ ||
vāsanāvaśataḥ prāṇaspandastena ca vāsanā |
kriyate cittabījasya tena bījāṅkurakramaḥ ||2||24-26||

The unsteady mind is the cause of birth, disease, old age and death, which are generated by Vāsanās i.e. thoughts resulting from stress. The influence of Vāsanās is to produce pulsation at the root of Prāṇā, which is carried to different body parts by the interconnected network of Chakras and Nāḍīs; from those, Vāsanās come again, like seed and sprout. For the tree of the human mind, pulsation and Vāsanā are two seeds – when one dies both die.

Muktikopaniṣad, Ch: 2 V: 24-26

From this we conclude that a disturbed mind will impair the flow of Prāṇā in the body. Balanced Prāṇā flow reflects a healthy mind, and will benefit a student's cognitive skills.

2.6 Prāṇāyāma:

2.6.1 The Remedy for Prāṇa Disturbances

अन्योऽन्तर आत्मा प्राणमयः । तेनैष पूर्णः ॥२।२।१॥

anyo'ntara ātmā prāṇamayah| tenaiṣa pūrṇah||2|2|1||

*There is a body subtler than the physical body called the Prāṇamaya Kośa,
which pervades the entire Annamaya Kośa.*

Taittirīya Upaniṣad, Ch: 2.2 V: 1

तस्मिन्सति श्वासप्रश्वासयोर्गतिविच्छेदः प्राणायामः ॥२।४९॥

tasminsati śvāsapraśvāsayorgativicchedaḥ Prāṇāyāmaḥ||2|49||

*(When āsana is established) Prāṇāyāma, the enlivenment of Prāṇā,
is attained by the steady alternation of inhalation and exhalation.*

Yogā Sūtras II.49

चित्तादिसर्वभावेषु ब्रह्मत्वेनैव भावनात् ।

निरोधः सर्ववृत्तीनां प्राणायामः स उच्यते ॥११८॥

niṣedhanaṁ prapañcasya recaḥkāhyaḥ samīraṇaḥ|

brahmaivāsmīti yā vṛttiḥ pūrako vāyurīritaḥ||118||

निषेधनं प्रपञ्चस्य रेचकार्यः समीरणः ।

ब्रह्मैवास्मीति या वृत्तिः पूरको वायुरीरितः ॥११९॥

cittādisarvabhāveṣu brahmatvenaiva bhāvanāt|

nirodhaḥ sarvavṛttinām Prāṇāyāmaḥ sa ucyate||118||

ततस्तद्वृत्तिनैश्चल्यं कुम्भकः प्राणसंयमः ।

अयं चापि प्रबुद्धानामज्ञानां घ्राणपीडनम् ॥१२०॥

tatastadṛṛttinaiścalyaṁ kumbhakaḥ prāṇasaṁyamah|

ayam cāpi prabuddhānamajñānām ghrāṇapīḍanam||120||

*Prāṇāyāma is the control of all life-forces by realizing naught but Brahman in all things
as the mind, etc. The negation of the universe is the outgoing breath. The thought 'I am
Brahman' itself is called the incoming breath. The permanence of that thought*

thereafter is the restrained breath. This is the Prāṇāyāma of the wise, while the pressing of the nose is only for the unknowing.

Aparokṣānubhūti, V: 118-120

रेचकः पूरकश्चैव कुम्भकः प्रणवात्मकः ।

प्राणायामो भवेदेवं मात्राद्वादशसंयुतः ॥१०१॥

recakaḥ pūrakaścaiva kumbhakaḥ praṇavātmakaḥ |
prāṇāyāmo bhavedevaṁ mātrādvādaśasamyutaḥ ||101||

मात्राद्वादशसंयुक्तौ निशाकरदिवाकरौ ।

दोषजालमबध्नन्तौ ज्ञातव्यौ योगिभिः सदा ॥१०२॥

mātrādvādaśasamyuktau niśākaradivākarau |
doṣajālamabadhnantau jñātavyau yogibhiḥ sadā ||102||

The inhalation, retention and exhalation are the Prāṇāyāma itself.

Prāṇāyāma should be practised like this for a number of twelve rounds.

*Twelve rounds through the ida and pingala nadis releases
the network of impurities. Yogis should always bear this in mind.*

Yogā Cuḍāmaṇi Upaniśad, V: 101-102

2.6.2 Method of Prāṇāyāma: Haṭh Yogā Pradipika describes in detail Prāṇā and its enlivenment by the method of Prāṇāyāma.

कृत्वा सम्पुटितौ करौ दृढतरं बद्ध्वा तु पद्ममासनं

गाढं वक्षसि सन्निधाय छिबुकं धयायंश्छ तच्छेतसि ।

वारं वारमपानमूर्ध्वमनिलं परोत्सारयन्पूरितं

नयञ्छन्प्राणमुपैति बोधमतुलं शक्तिप्रभावान्नरः ॥१॥४८॥

kṛtvā sampuṭitau karau dṛḍhatarāṁ baddhvā tu padmamāsanāṁ
gāḍhaṁ vakṣhasi sannidhāya chibukaṁ dhayāyaṁścha tachchetasi |
vāraṁ vāramapānamūrdhvamanilāṁ parotsārayanpūritāṁ
nayañchanprāṇamupaiti bodhamatulaṁ śaktiprabhāvānnaraḥ ||1||48||

*After placing both hands together in the lap, seated firmly in Padmāsana,
pressing the chin fixedly to the chest, and dwelling on Him in the mind,*

joining the prāna and apāna in the navel by drawing the apāna-vāyū up (performing Mūla Bandha) and pushing down the inhaled breath, (the Yogi) attains the highest intelligence by thus awakening the (kundalini) śakti.

Hath Yogā Pradipika, Ch: 1 V: 48.

When Apāna Vāyu is drawn gently up and after filling in the lungs with the air from outside, the Prāṇā is forced down so as to join both of them in the navel, they then enter the Kuṇḍilinī together, and, reaching the Brahmarandhra (the great hole), they make the mind calm. Then the mind can contemplate the nature of Atman, and enjoy the highest bliss.

2.6.3 Benefits of Prāṇāyāma

ततः क्षीयते प्रकाशावरणम् ॥२॥५२॥

tataḥ kṣīyate prakāśāvaraṇam ||2||52||

Thereby whatever is covering the light vanishes.

Yogā Sūtras II.52

In Yogā Sūtras, Mahārṣi Patañjali describes this potential of Prāṇāyāma to activate dormant centers in the brain. As these dormant potentials are enlivened, the practitioner is propelled into greater understanding of the heights and depths of his being, beyond those of ordinary experience.

विधिवत्प्राणसंयामैर्नाडीचक्रे विशोधिते ।

सुषुम्नावदनं भित्त्वा सुखाद्विशति मारुतः ॥२॥४१॥

vidhivatprāṇasaṁyāmairnāḍīcakre viśodhite |
suṣumnāvadanam bhittvā sukhādvīśati mārutaḥ ||2||41||

By systematically retaining the Prāṇā, the Nādis and Chakras are purified.

The Prāṇā thus opens the doorway to the Sushumna and easily enters it.

Hath Yogā Pradīpikā II.41

चले वाते चलं चित्तं निश्चले निश्चलं भवेत् ।

योगी स्थाणुत्वमाप्नोति ततो वायुं निरोधयेत् ॥२॥२॥

cale vāte calaṁ cittam niścale niścalaṁ bhavet |
yogī sthāṇutvamāpnoti tato vāyum nirodhayet ||2||2||

Disturbing respiration disturbs the mind.

Steadying respiration enables the Yogī to steady the mind.

Haṭh Yogā Pradīpikā II.2

यथा पर्वतधातुनां दह्यन्ते धमनन्मलाः ।

तथेन्द्रियकृता दोषा दह्यन्ते प्राणधारणात् ॥७॥

yathā parvatadhātunām dahyante dhamananmalāḥ |
tathendriyakṛtā doṣā dahyante prāṇadhāraṇāt ||7||

*Just as impurities in mountain minerals are burnt away by the blower,
so stains caused by sense organs are consumed by controlling the Prāṇā.*

Amṛtānanda Upaniśad, V: 7

पूर्वार्जितानि कर्माणि प्राणायामेन निश्चितम् ।

नाशयेत्साधको धीमानिहलोकोद्भवानि च ॥३॥४९॥

pūrvārjitāni karmāṇi prāṇāyāmena niścitama |
nāśayetsādhako dhīmānihalokodbhavāni ca ||3||49||

*By Prāṇāyāma, wise practitioners surely destroy all their karma,
whether acquired in this life or past lives.*

Śiva Samhitā III.49

पापतूलचयानाहोप्रदहेत्प्रलयाग्निना ।

ततः पापविनिर्मुक्तः पश्चात्पुण्यानि नाशयेत् ॥३॥५१॥

pāpatūlacayānāhopradahetpralayāgninā |
tataḥ pāpavinirmuktaḥ paścātpuṇyāni nāśayet ||3||51||

As the fire of annihilation burns away everything,

Prāṇāyāma destroys the multitude of sins.

After freeing the yogi from all his sins,

it then destroys (consequences of) all his virtuous actions.

Śiva Saṁhitā, Ch III, V 51

Śiva Saṁhitā chapter III, verses 7-14 give details of different Prāṇāyāma practices.

After saluting the lords of Yogā and sipping a little water, (the aspirant) should begin his Prāṇāyāma exercises, according to the words of Śiva described in the Kūrma Purāṇa. (Verse 7).

(Using phrases like) "Salutations to the lords of Yogā and their disciples, and Guru Vināyaka, the Yogī should unite with me with composed mind." (Verse 8).

When practicing, he should sit in Siddhāsana, and having performed bandha and Kumbhaka, he should begin with 10 rounds of Prāṇāyāma on the first day, and increase by 5 rounds daily. (Verse 9).

With composed mind 80 Kumbhakas should be performed at a time; beginning first with the Candra (the left nostril) and then Sūrya (the right nostril). (Verse 10).

Wise men call this practice Aṅuloma - Viloma. Having practiced Sūrya Bhedana, with Bandhas, (the wise) should practice Ujjayī, Śitalī, Śitakārī and Bhastrikā, other practices are optional. (Verses 11-12).

He should practise mudrās properly, as instructed by his guru. Then sitting with Padmāsana, he should listen attentively to Anāhata Nāda (the Cosmic Om heard in the Hṛdaya). (Verse 13).

He should *reverently offer the fruits of all his practice to God*. On completion of his practice, a warm bath should be taken. (Verse 14).

2.6.4 Nāḍī Śodhana

बद्धपद्मसना योगी प्राणं चंद्रेण पूरयेत् ।

धारयित्वा यथाशक्ति भूयः सूर्येण रेचयेत् ॥२।७॥

प्राणं सूर्येण चाकृष्य पूरयेदुदरं शनैः ।

विधिवत्कुंभकं कृत्वा पुनश्चंद्रेण रेचयेत् ॥२।८॥

baddhapadmasanā yogī prāṇam candreṇa pūrayeta |
dhārayitvā yathāśakti bhūyaḥ sūryeṇa recayet ||2|7||
prāṇam sūryeṇa cākṛṣya pūrayedudaram śanaiḥ |
vidhivatkuṃbhakam kṛtvā punaścandreṇa recayet ||2|8||

Sitting in Padmāsana the Yogī should inhale through the Chandra (left) nostril (closing the right one); and, after retaining it according to his ability, he should expel it slowly through the Sūrya (right) nostril (closing the left one). Then, drawing in the air through the Sūrya (right nostril) slowly, the belly should be extended (i.e. abdominal breathing), and after performing Kumbhaka as before, it should be expelled slowly through the Chandra (left nostril).

Haṭh Yogā Pradīpikā II.7-8.

येन त्यजेत्तेन पीत्वा धारयेदतिरोधतः ।

रेचयेच्च ततोऽन्येन शनैरेव न वेगतः ॥२।९॥

yena tyajettena pītvā dhārayedatirodhataḥ |
recayecca tato'nyena śanaireva na vegataḥ ||2|9||

Inhaling thus through the (nostril), through which it was expelled, and having retained it, to the extent possible, it should be exhaled through the other (nostril), slowly and not forcibly.

Haṭh Yogā Pradīpikā II.9.

प्राणं छेद इडया पिबेन्नियमितं भूयोऽन्यया रेचयेत्

पीत्वा पिंगलया समीरणमथो बद्ध्वा त्यजेद्दामया ।

सूर्याचन्द्रमसोरनेन विधिनाभ्यासं सदा तन्वतां

शुद्धा नाडिगणा भवन्ति यमिनां मासत्रयादूर्ध्वतः ॥२।१०॥

prāṇam cheda iḍayā pibenniyamitam bhūyo'nyayā recayet

pītvā piṅgalayā samīraṇamatho baddhvā tyajedvāmayā |
sūryācandramasoranena vidhinābhyāsaṁ sadā tanvatām
śuddhā nāḍigaṇā bhavanti yaminām māsatrāyādūrdhvataḥ ||2|10||

Air inhaled through the left nostril should be expelled again through the other; then inhaling through the right nostril, and retaining it, it should be exhaled through the left nostril. Practicing in this way, alternately through right and left nostrils, for 3 months or more cleanses the practitioner's entire network of nāḍīs, i.e. frees them from all impurities.

Haṭh Yogā Pradīpikā, II.10

प्राणायामेन युक्तेन सर्वरोगक्षयो भवेत् ।

अयुक्ताभ्यासयोगेन सर्वरोगसमुद्भवः ॥२॥१६॥

prāṇāyāmena yuktena sarvarogakṣayo bhavet |
ayuktābhyāsayogena sarvarogasamudbhavaḥ ||2|16||

*By correct Prāṇāyāma practises, all diseases are eradicated;
In contrast, improper practice can generate any disease.*

Haṭh Yogā Pradīpikā, II.16

हिक्का श्वासश्च कासश्च शिरःकर्णाक्षिवेदनाः ।

भवन्ति विविधा रोगाः पवनस्य प्रकोपतः ॥२॥१७॥

hikkā śvāsaśca kāsaśca śiraḥkarṇākṣivedanāḥ |
bhavanti vividhā rogāḥ pavanasya prakopataḥ ||2|17||

*Hiccough, asthma, cough, pain in the head, the ears, and the eyes;
these and diseases of various other kinds are generated by disturbance of the breath.*

Haṭh Yogā Pradīpikā, II.17

अधस्तात्कुंचनेनाशु कंठसंकोचने कृते ।

मध्ये पश्चिमतानेन स्यात्प्राणो ब्रह्मनाडिगः ॥२॥१४६॥

adhastātkuñcanenāśu kaṇṭhasaṅkocane kṛte |
madhye paścimatānena syātpraṇo brahmanāḍigaḥ ||2|46||

Drawing up from below (Mūla Bandha,) and contracting the throat (Jālandhara Bandha) and by pulling back the middle of the front portion of the body (Uḍḍīyāna Bandha), forces the Prāṇā up the Brahma nāḍīs (the Suṣumnā).

Hatḥ Yogā Pradīpikā, II.46

2.6.5 Sūrya Bhedana

आसने सुखदे योगी बद्ध्वा चैवासनं ततः ।

दक्षनाड्या समाकृष्य बहिःस्थं पवनं शनैः ॥२॥४८॥

āsane sukhade yogī baddhvā caivāsanam tataḥ |
dakṣanāḍyā samākṛṣya bahiḥstham pavanam śanaiḥ ||2|48||

Seated in any comfortable position, and adopting that āsana, the Yogī should draw in the air slowly, through the right nostril.

Hatḥ Yogā Pradīpikā, II.48

आकेशादानखाग्राच्च निरोधावधि कुंभयेत् ।

ततः शनैः सव्यनाड्या रेचयेत्पवनं शनैः ॥२॥४९॥

ākeśādānakhāgrācca nirodhāvadhi kumbhayet |
tataḥ śanaiḥ savyanāḍyā recayetpavanam śanaiḥ ||2|49||

Then retaining it, as if from the nails to the tips of the hair, it should then be let slowly out through the left nostril slowly.

Hatḥ Yogā Pradīpikā, II.49

N.B. This should be done using both nostrils, inhaling alternately through one, and exhaling through the other.

कपालशोधनं वातदोषघ्नं कृमिदोषहृत् ।

पुनःपुनरिदं कार्यं सूर्यभेदनमुक्तमम् ॥२॥५०॥

kapālaśodhanam vātadoṣaghnam kṛmidoṣahr̥t |
punaḥpunaridaṁ kāryam sūryabhedanamuktamam ||2|50||

This excellent Sūrya Bhedana cleanses the forehead (frontal sinuses),

removes Vāta disorders, and worms, and should therefore be performed repeatedly.

Haṭh Yogā Pradīpikā, II.50

N.B. Although the word ‘Prāṇāyāma’ should be translated as ‘enlivenment of Prāṇā’ rather than ‘controlling the Prāṇā’, these techniques described here definitely involve ‘controlling the breathing’. But that does not justify translating the word, ‘Prāṇāyāma’ as ‘breath control’.

2.6.6 Bhastrikā Prāṇāyāma

यथा लगति हृत्कंठे कपालावधि सस्वनम् ।

वेगेन पूरयेच्चापिहृत्पद्मावधि मारुतम् ॥२॥६१॥

yathā lagati hṛtkanṭhe kapālāvadhi sasvanam |
vegena pūrayeccāpihṛtpadmāvadhi mārutam ||2||61||

(The body) should be filled up to the lotus of the heart, by drawing it in with force, making noise and touching the throat, the chest and the head.

Haṭh Yogā Pradīpikā II.61

पुनर्विरेचयेत्तद्वत्पूरयेच्च पुनः पुनः ।

यथैव लोहकारेण भस्त्रा वेगेन चाल्यते ॥२॥६३॥

punarvirecayettadvatpūrayecca punaḥ punaḥ |
yathaiva lohakāreṇa bhastrā vegena cālyate ||2||62||
*Again and again the breath should be expelled and filled as before,
just as a blacksmith’s bellows are worked.*

Haṭh Yogā Pradīpikā II.62

तथैव स्वशरीरस्थं चालयेत्पवनं धिया ।

यदा श्रमो भवेद्देहे तदा सूर्येण पूरयेत् ॥२॥६३॥

tathaiva svaśarīrasthaṁ cālayetpavanaṁ dhiyā |
yadā śramo bhaveddehe tadā sūryeṇa pūrayet ||2||63||

*In the same way, the air of the body should be moved intelligently,
filling it through Sūrya when fatigue is experienced.*

Haṭh Yogā Pradīpikā II.63

यथोदरं भवेत्पूर्णमनिलेन तथा लघु ।

धारये न्नासिकां मध्यातर्जनीभ्यां विना दृढम् ॥२॥६४॥

yathodaram bhavetpūrṇamanilena tathā laghu |
dhāraye nnāsikāṁ madhyātarjanībhyāṁ vinā dṛḍham ||2||64||

*The air should be drawn in through the right nostril by pressing the thumb against the
left side of the nose, so as to close the left nostril; when filled to the full, the nostril
should be closed with the ring finger (adjacent to the little finger) and the breath
retained.*

Haṭh Yogā Pradīpikā II.64

*(The practice is described using the left hand; some say that females should use the
right hand.)*

विधिवत्कुंभकं कृत्वा रेचयेदिडयानिलम् ।

वातपित्तश्लेष्महरं शरीराग्निविवर्धनम् ॥२॥६५॥

vidhivatkumbhakaṁ kṛtvā recayedidayānilam |
vātapittaśleṣmaharam śarīrāgnivivardhanam ||2||65||

Having retained it appropriately, it should be exhaled through the Idā (left nostril).

This reduces Vāta, Pitta and Kapha (imbalances), and increases digestive power.

Haṭh Yogā Pradīpikā II.65

सम्यग्गात्रसमुद्भूतग्रंथित्रयविभेदकम् ।

विशेषेणैव कर्तव्यं भस्त्राख्यं कुंभकं त्विदम् ॥२॥६७॥

samyaggātrasamudbhūtagranthitrayavibhedakam |
viśeṣeṇaiva kartavyaṁ bhastrākhyāṁ kumbhakantvidam ||2||67||

This form of Bhastrikā is to be done dutifully, for it releases the knots closing the chakras: (Brahma granthis for the first two, Viṣṇu granthi for the second two, and Rudra granthi for nos 5 & 6).

Haṭh Yogā Pradīpikā II.67

2.6.7 Bhrāmari Pranāyāma

वेगाद्धोषं पूरकं भृंगनादं भृंगीनादं रेचकं मंदमंदम् ।

योगीन्द्राणामेवमभ्यासयोगाच्चित्ते जाता काचिदानंदलीला ॥२।६८॥

vegādghoṣam pūrakam bhṛṅganādam bhṛṅginādam recakam
mandamandam |

yogīndrāṇāmevamabhyāsayogāccitte jātā kācidānandalīlā ||2|68||

Filling the lungs forcefully, making a bee-like noise, and then expelling it slowly in the same way; this practice causes the playful ecstasy in the minds of masters of Yogā.

Haṭh Yogā Pradīpikā II.68

प्राणायामस्त्रिधा प्रोक्तो रेचपूरककुम्भकैः ।

सहितः केवलश्चेति कुम्भको द्विविधो मतः ॥२।७१॥

prāṇāyāmastridhā prokto recapūrakakumbhakaiḥ |
sahitaḥ kevalaśceti kumbhako dvividho mataḥ ||2|71||

Considering Pūraka (Filling), Rechaka (expelling) and Kumbhaka (confining), Prāṇāyāma is of three kinds, but considering it accompanied by Pūraka and Rechaka, and without these, it is of two kinds only, i.e., Sahita (with) and Kevala (alone).

Haṭh Yogā Pradīpikā II.71

2.7 VEDIC MATHS

The 1992 posthumous book, ‘Vedic Maths’, by Swami Bharati Krishna Tirtha Maharaj (1884-1960) comprises 29 Sūtras, 16 main Sūtras and 13 Upasūtras given below. The Sūtras are entirely constituted from terms traceable to Sthapatyaveda, the Upaveda of

Atharvaveda, with its Śulba Sūtras and Śrota Sūtras used to building altars (see Table 1). The name has provoked controversy (Dani, 1993), but it employs Vedic-style aphorisms (Kansara, 2000).

TABLE 1: TERMS USED IN THE VEDIC MATHS SŪTRAS AND UPASŪTRAS AND THEIR REFERENCES (Kansara, 2000)

Anta	Bodhāyana Śulba Sūtras I. 23.
Antya	Taittiriya Samhitā 1. 7. 9. 1.; Maitreya Samhitā 1.11.3; "the last member of a mathematical series" - Monier-Williams Sanskrit-English Dictionary, p. 44, Col. 3: Āpastamba Śulba Sūtras 2.3; Atharvaveda Anukramaṇikā 20.9; 104; 118.
Adya	Atharvaveda Samhitā 19.22.1.
Anurupya / Anurupatva	Āpastamba Śulba Sūtras 13.8.
Urdhva/ Urdhvapramāṇa	Āpastamba Śulba Sūtras 9.15; Bodhāyana Śulba Sūtras 2.3; Vadhulā Śulba Sūtras 7.2; 11. 9.; Hiranyakeśi Śulba Sūtras 3.14; 4.20.
Ekanyūna / Ekona	Kātyāyana Śulba Sūtras 6.7.
Ekādhika	Kātyāyana Śulba Sūtras 6.7.
Guṇaka / Guṇa	Bodhāyana Śulba Sūtras 1.30.
Guṇita	Mādhyandina Samhitā. 16.2.; Yājñavalkya Smṛti 2.104-105.
Carama	Bodhāyana Śrota Sūtras 10.48.2.
Tiryak / Tiryag-Dviguṇa	Kātyāyana Śulba Sūtras 6.7
Tiryag-Bheda	Bodhāyana Śulba Sūtras 17.8; 19.7
Tiryan-Mana	Kātyāyana Śulba Sūtras 7.32; Bodhāyana Śulba Sūtras 1.46.
Daśaka	Atharvaveda Anukramaṇikā 19.17.
Daśataḥ	Āśvalāyana Śrota Sūtras 8.5.7; Āpastamba Śrota Sūtras 20.14.2; 22.17.5; Hiranyakeśi Śrota Sūtras 17.6.42. (15)
Navataḥ	Mādhyandina Śrota Sūtras 8.5.7; Bodhāyana Śrota Sūtras 15.23.
Nikhila	Mādhyandina Śrota Sūtras 1.7.8.10.
Pūraṇa	Kātyāyana Śrota Sūtras 24.7.18; Āpastamba Śrota Sūtras 16.26.9; 16.27.6;
Pūrayet	Bodhāyana Śulba Sūtras 8.12
Madhyama	Sāṅkhyāyāna Śrota Sūtras 7.27.20;

Madhyama-Pūrva	Bodhāyana Śulba Sūtras 7.17. (20)
Yāvat....Tāvat – Yāvat - Pramāṇa	Kātyāтана Śulba Sūtras 37.12; Āpastamba Śulba Sūtras 3.11; Hiraṇyakeśi Śulba Sūtras 1.50;
Tavad-Antarāla	Bodhāyana Śulba Sūtras 8.11; Hiraṇyakeśi Śulba Sūtras 3.17.
Yojayet / Yujyate	Hiraṇyakeśi Śulba Sūtras 4.20.; Āpastamba Śulba Sūtras 11.46; Bodhāyana Śulba Sūtras 21.7.
Lopāna / Lopa	Kātyāтана Śrota Sūtras 19.7.6.
Varga	Kātyāтана Śulba Sūtras 3.7; Āpastamba Śulba Sūtras 3.11; Hiraṇyakeśi Śulba Sūtras 1.50.
Veṣṭhana	Sāṅkhyāyāṇa Grhya Sūtra. 3.1.8.
Vyaṣṭhi	Āpastamba Śrota Sūtras 10.6; Sāṅkhyāyāṇa Śrota Sūtras 16.1.1.
Śūnya	Drāhyāyāṇa Śrota Sūtras 4.4.22.
Śeṣa	Kātyāтана Śulba Sūtras 3.2.3; Āpastamba Śulba Sūtras 2.15; 13.2; Bodhāyana Śulba Sūtras 2.10.12.; Hiraṇyakeśi Śulba Sūtras 1.36; 4.34.
Samaṣṭhi	Bodhāyana Śrota Sūtras. 17.13.7.
Sāmyasamuccaya	Kātyāтана Śrota Sūtras 1.8.7. 21; 14. 3.5.
Sopāntya / Upāntya	Āpastamba Śulba Sūtras 1.12; Hiraṇyakeśi Śulba Sūtras 1.22.

Caption: Table 1 lists all the words used in the 16 Sūtras and 13 Upasūtras of Vedic Maths, giving precise references for each in Vedic texts. This establishes the Vedic quality of the work.

2.7.2 The 16 Sūtras

एकाधिकेन पूर्वेण

ekādhikena pūrveṇa

By one more than the one before

निखिलं नवतश्चरमं दशतः

nikhilam navataścaramam daśataḥ

All from 9 and the last from 10

ऊर्ध्वतिर्यग्भ्याम्
Ūrdhvatiryagbhyām
Vertically and Crosswise

परावर्त्य योजयेत्
parāvartya yojayet
Transpose and Apply

शून्यं साम्यसमुच्चये
śūnyaṁ sām्यasamuccaye
If the Samuccaya is the same, it is Zero

आनुरुप्ये शून्यं अन्यत्
ānurupye śūnyaṁ anyat
If one is in ratio the other is zero

संकलनव्यवकलनाभ्यां
saṅkalanayavakalanābhyām
By Addition and by Subtraction

पूर्णपूर्णाभ्याम्
Pūrṇapūrṇābhyām
By Completion or Non-Completion

चलनकलनाभ्याम्
calanakalanābhyām
Differential Calculus

यावदूनं
yāvadūnaṁ
By the Deficiency

व्यष्टिसमष्टिः

vyāṣṭisamaṣṭiḥ

Specific and General

शेषान्क्येन चरमेण

śeṣānkyena carameṇa

The Remainders by the Last Digit

सोपान्त्यद्वयमन्त्यं

sopāntyadvayamantyaṃ

The Ultimate and Twice the Penultimate

एकन्यूनेन पूर्वेण

ekanyūnena pūrveṇa

By One Less than the One Before

गुणितसमुच्चयः

guṇitasamuccayaḥ

The Product of the Sum

गुणकसमुच्चयः

guṇakasamuccayaḥ

All the Multipliers

2.7.3 The Upasūtras (Corollaries)

आनुरूप्येण

ānurupyēṇa

Proportionately

शिष्यते शेषसंज्ञः

śiṣyate śeṣasañjñāḥ

The Remainder Remains Constant

आद्यमाद्ये नान्त्यमन्त्येन
ādyamādye nāntyamantyaena
The First by the First and the Last by the Last

केवलैः सप्तकं गुण्यात्
kevalaiḥ saptakam guṇyāt
For 7 the Multiplicand is 143

वेष्टनम्
veṣṭanam
By Osculation

यावदूनं तावदुनं
yāvadūnam tāvadunam
Lessen by the Deficiency

यावदूनं तावदूनीकृत्य वर्गं च योजयेत्
yāvadūnam tāvadūnikṛtya vargam ca yojayet
Whatever the deficiency lessen by that amount and set up the Square of the deficiency

अन्त्ययोर्दशकेऽपि
antyayordaśake'pi
Last Totaling 10

अन्त्ययोरेव
Antyayoreva
Only the Last Terms

समुच्चयगुणितः
samuccayaguṇitaḥ
The Sum of the Products

लोपनस्थापनाभ्यां

lopanasthāpanābhyām

By Alternate Elimination and Retention

विलोकनं

vilokanam

By Mere Observation

गुणितसमच्चयः समुच्चयगुणितः

guṇitasamaccuyah samuccayaguṇitah

The Product of the Sum is the Sum of the Products

ध्वजाङ्क

dhvajāṅka

On the Flag

2.7.4 Illustrative applications of some of the Sūtras

1. **Sūtra 1** : By one more than one before

$$35^2 = 3 \times 4 / 25 = 1225$$

2. **Sūtra 2** : All from 9 and the last from 10

$$\begin{array}{r} 97 - 03 \\ \times 94 - 06 \end{array}$$

Ans: 9118

3. **Sūtra 3**: Vertically and crosswise

$$\begin{array}{r} \times \quad \begin{array}{|c|c|} \hline 3 & 2 \\ \hline 2 & 3 \\ \hline \end{array} \\ \\ 1 \\ \\ 736 \end{array}$$

4. **Sūtra 4** : Transpose and apply

$$(x - 3) \cdot (x - 2) = (x + 1)(x + 2)$$

$$x = \frac{1(2) - (-3)(-2)}{-3 - 2 - 1 - 2}$$

5. **Sūtra 5** : If the Samuccaya is the same then it is zero

$$\frac{1}{3x-2} + \frac{1}{2x-1} = 0 \text{ Find } x.$$

Ans: If the numerators are same then

Sum of the denominators = 0

$$5x - 3 = 0 \Rightarrow x = \frac{3}{5}$$

6. **Sūtra 6** : If the one is in ratio the other is zero

solve $3x + 7y = 2$

$$4x + 21y = 6$$

Ans: Observe that y-coefficients are in the ratio 7:21 or 1:3 which is same as the ratio of independent terms which is 2:6. Hence the other variable $x=0$ and $7y=2$

$$\text{or } y = \frac{2}{7}$$

7. **Sūtra 7**: By addition and subtraction

solve $1955x - 476y = 2482$

$$476x - 1955y = -4913$$

Here just add, $2431(x - y) = -2431 \Rightarrow x - y = -1$

Subtract, $1479(x + y) = 7395 \Rightarrow x + y = 5$

Once again add $2x=4$ or $x=2$

Subtract $-2y=-6$, $y=3$

8. Sūtra 8 : By the completion or non-completion

$$\text{solve } x^3 + 6x^2 + 11x + 6 = 0$$

$$\text{Since } (x + 2)^3 = x^3 + 6x^2 + 12x + 8$$

Add $(x + 2)$ both the sides we get

$$x^3 + 6x^2 + 11x + 6 + x + 2 = x + 2$$

$$\Rightarrow x^3 + 6x^2 + 12x + 8 = x + 2$$

$$(x + 2)^3 = (x + 2)$$

this is of the form $y^3 = y$ for $y = x + 2$

$$\text{solution: } y = 0, y = 1, y = -1$$

$$\Rightarrow x + 2 = 0, 1, -1 \Rightarrow x = -2, -1, -3$$

9. Sūtra 9 : Differential Calculus

$$\text{Solve } x^2 - 7x + 10 = 0$$

$$f'(x) = \pm\sqrt{D}$$

$$2x - 7 = \pm\sqrt{9} \Rightarrow 2x - 7 = 3, 2x - 7 = -3$$

$$\Rightarrow x = 5, 2$$

10. Sūtra 10: By the deficiency

$$112^2 = 124_144 = 12544 \quad (\text{First part is } 112+12=124, \text{ second part is } 12^2 = 144)$$

11. Sūtra 11: Individuality or Totality

$$\text{If } (x + 7)^4 + (x + 5)^4 = 706, \text{ Find the value of } x.$$

Let $x + 6$ the average of the two binomials = a

$$\therefore (a + 1)^4 + (a - 1)^4 = 706$$

\therefore owing to the cancellation of the odd powers x^3 and x ,

$$2a^4 + 12a^2 + 2 = 706, \therefore a^4 + 6a^2 - 352 = 0$$

$$\therefore a^2 = 16 \text{ or } -22, \therefore a = \pm 4 \text{ or } \pm\sqrt{-22}, \therefore x = -2 \text{ or } -10 \text{ or } \pm\sqrt{-22} - 6.$$

12. Sūtra 12 : The remainders by the last digit

We can write the answers for the following fractions just by observing first two fractions. The pattern shows geometric progression with common ratio $\frac{1}{2}$.

$$\frac{65}{2} = 32\frac{1}{2}, \frac{65}{4} = 16\frac{1}{4}, \frac{65}{8} = 8\frac{1}{8}, \frac{65}{16} = 4\frac{1}{16}, \frac{65}{32} = 2\frac{1}{32}$$

13. Sūtra 13 : The ultimate and twice the penultimate

If $\frac{1}{AB} + \frac{1}{AC} = \frac{1}{AD} + \frac{1}{BC}$, and A,B,C,D are in A.P. then $D+2C=0$ is one solution.

$$\text{Exa: } \frac{1}{x^2+5x+6} + \frac{1}{x^2+6x+8} = \frac{1}{x^2+7x+10} + \frac{1}{x^2+7x+12}$$

$$\Rightarrow \frac{1}{(x+2)(x+3)} + \frac{1}{(x+2)(x+4)} = \frac{1}{(x+2)(x+5)} + \frac{1}{(x+3)(x+4)}$$

$$\Rightarrow x + 5 + 2(x + 4) = 0 \Rightarrow 3x = -13 \Rightarrow x = \frac{-13}{3}$$

14. Sūtra 14 : By the one less than one before

$$878 \times 9999 = 8779122$$

Here $877 = 878-1$ and complement of 0877 is 9122

15. Sūtra 15 : The product of the sum

This is used to verify the factors of an expression.

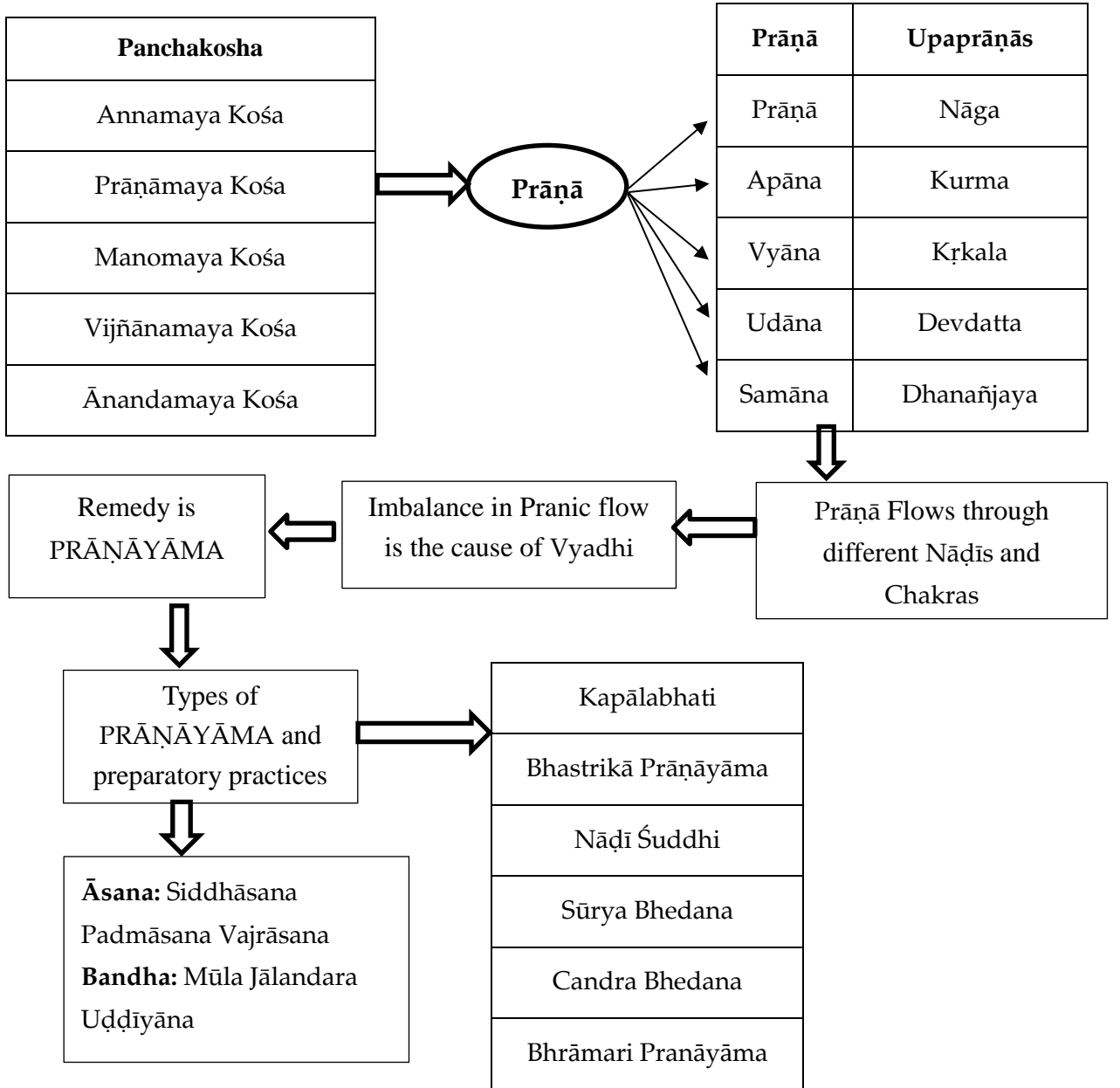
$$x^2 + 5x + 6 = (x + 3)(x + 2)$$

sum of co-efficient in $1 + 5 + 6 = 12$

Product of sum of coefficients of factors = $(1+3)(1+2)=4 \times 3=12$

2.8 SUMMARY

Prāṇā and Prāṇāyāma



Vedic Maths:

Swami Bharati Krishna Tirtha Maharaj (1884-1960) formulated 29 Sūtras, 16 main Sūtras and 13 Upasūtras and named as Vedic Maths. Sūtras are in Vedic Style and some of the words are taken from Śulba Sūtras. They are suitable for high school mathematics calculations.