CHAPTER – 3

REVIEW OF SCIENTIFIC LITERATURE

3.0 REVIEW OF SCIENTIFIC RESEARCH LITERATURE

3.1 MATH ANXIETY

Mathematics Anxiety has been defined as “a feeling of tension and anxiety that interferes with the manipulation of numbers and the solving of mathematical problems in ordinary life and academic situations” (Richardson et al., 1972). Ashcraft and Kirk define Math Anxiety as — “a feeling of tension, apprehension, or fear that interferes with math performance" (Ashcraft et al., 2001). As earlier thought, mathematical learning difficulty does not always result from cognitive difficulties. A substantial number of children and adults have mathematics anxiety, which may severely disrupt their mathematical learning and performance, causing avoidance of mathematical activities, and overloading and disrupting working memory during mathematical tasks (Dowker et al., 2016).

When Math Anxiety occurs, it brings about different effects which are either mental or emotional. Symptoms of mathematical anxiety include “high test anxiety, low enjoyment of math, low self-confidence in math, lack of motivation toward math, negative attitudes toward math teachers, avoidance of math classes, and low achievement in math classes” (Hembree, 1990; Jameson, 2014; Ma, 1999). Mentally, the operation of the working memory system is hindered, therefore disrupting the mathematical processing that relies on the working memory (Ashcraft et al., 1998). In a study conducted by Hunt, Clark-Carter, and Sheffield (2011), math anxious people were more likely to be conscious of what others might think, leading to panic. Almost 90% of participants thought, while solving problems, that they might make mistakes. Almost half were concerned about the amount of time remaining, and worried about methods needed for a problem. Hunt, Clark-Carter, and Sheffield (2015) conclude that Math Anxiety causes a higher error rate on math problems.

Math Anxiety strikes people who commonly find dealing with everyday activities involving mathematics difficult (Mark H Ashcraft et al., 1998). Compounding the problem, such people feel as if alone in their situation (Sutter, 2006), making them fear being judged lacking in ability, should they ask for help (Sutter, 2006).

3.1.1 Prevalence of Math Anxiety

Different papers report various percentages of Math Anxiety, from 11% to 68%. Richardson and Suinn (1972) estimate that 11% of university students show high levels of mathematics anxiety whereas, Betz (1978) concluded that about 68% of students enrolled in mathematics classes experience high mathematics anxiety. M H Ashcraft & Moore (2009) reported a 17% estimate of the proportion of the population with elevated Math Anxiety. Johnston-Wilder, Brindley and Dent (2014) found that about 30% of a group of apprentices showed high Math Anxiety, with a further 18% less affected. Beilock and Willingham (2014) found that Math Anxiety affect nearly 50% of US grade-school children. In English secondary school students on other hand, Chinn (2009) obtained the far lower value of 2–6%.

In another study, around one-third of the participants reported anxiety concerning ability to do math, some 20% about answering teachers’ questions, and 10% about doing math (Sorvo et al., 2017).

In the 2012 Programme for International Student Assessment (PISA), 59% of students averaged across 65 countries reported worrying about possible difficulties during mathematics classes, while 30% felt helpless when trying to solve a mathematics problem (OECD, 2013). A study at the Philippine University of Asia and the Pacific (Mejia, 2015), found 46% answering ‘Yes’ to the question asking whether mathematics made them anxious.

Math Anxiety is higher in females than males (Devine et al., 2018). Because of Math Anxiety, attitudes to mathematics deteriorate with age, with females tending to score higher on Math Anxiety, compared to males (Dowker et al., 2016). However, a recent study has found that females exhibit more general anxiety, while males have more Math Anxiety (Abraham et al., 2017). Another trans-national study confirmed this, finding higher Math Anxiety in females than males in 56 of the 64 countries participating in the Program for International Student Assessment (PISA) (Foley et al., 2017).

Research finds that math anxious adults have been so since they were young. Lazarus (1974), states that the “roots of Math Anxiety are in the elementary and secondary grades” (Mejia, 2015). Symptoms of Math Anxiety are evident in sixth grade students and continue to increase up to ninth grade (Mark H Ashcraft et al., 1998). Most of these studies found very high prevalence of Math Anxiety in the world.

3.1.2 Effects of Math Anxiety

3.1.2 A Performance

Daily maths lessons may start causing student anxiety in elementary school (Ramirez et al., 2013), slowly becoming a dominant feature of school experience (Maloney et al., 2010; Taylor et al., 2013), and causing poor performance, especially in mathematics. Compared to reading problems, Mathematics seems to evoke stronger emotional reactions, with anxiety particularly compromising performance (Punaro et al., 2012). Since exam performance determines career options, math anxiety can also play a major role in limiting career choice. As Tobias et al. (1980) observed, eliminating math from the curriculum automatically shuts students off from acceptance into most university majors, significantly narrowing a student’s options.

As is well-documented, Math Anxiety is accompanied by lower maths performance. Math Anxiety is inversely related to measures of maths achievement; math-anxious students reportedly obtain lower grades in mathematics than classmates with less Math Anxiety. The question of which aspects of mathematical performance are affected most is an ongoing topic research. Maloney et al., 2010b) observed that math-anxious undergraduates underperform on trivial problems, counting numbers of dots in sets of dots slower than less math-anxious classmates. Similarly, math-anxious students exhibit delayed response times in symbolic magnitude comparison tasks, where participants have to decide whether a given number is larger or smaller than a reference number (Maloney et al., 2011). Finally, Wang et al. (2014) showed that Math Anxiety and the accuracy with which adolescents and adult students estimate the position of a given number on a number line, are inversely related. These results suggest that Math Anxiety is related to individual differences in basic numerical skills in university students.

Ashcraft and Faust (1994) assessed the way Math Anxiety is related to arithmetic problem solving, asking students to solve small (e.g. 3+8=11) and larger (e.g., 9×16=144) arithmetic problems. Small problems are solved by fact retrieval from memory, just “popping up in one's mind”; solving larger problems requires application of some transformation or procedure (Siegler et al., 1996); e.g. the problem 9×16 can be solved by decomposing the problem into 9×10 and 9×6, and then summing up the solutions of the two easier calculations (i.e., 90+54=144). Math-anxious students showed lower overall performance in arithmetic (i.e., slower response times and higher error rates) than less math-anxious classmates. Performance differences were marginal for small arithmetic problems, but more pronounced for large problems, indicating that procedural strategies are especially affected by Math Anxiety. This was corroborated by another study, in which math-anxious students found arithmetic problems requiring a carry-over operation particularly difficult (Faust et al., 1996).

Recently, Lee and Cho (2017) have reported that Math Anxiety is associated with slower solution rates for large arithmetic problems, but not with retrieval of arithmetic facts. Interestingly, Ramirez et al., (2016) showed that having to use an advanced problem solving strategy partly caused Math Anxiety during arithmetic performance by elementary school children. Math-anxious children used more advanced strategies like decomposition to solve arithmetic problems less than their less math-anxious peers. This is in line with the findings reviewed above that Math Anxiety is related to arithmetic problem solving in adults; especially when solving a problem requires application of an arithmetic procedure.

Since the majority of research on Math Anxiety has been conducted with high school and college students, math performance was mostly assessed by students' grades or age-related mathematics achievement tests assessing higher-order mathematics (Betz, 1978b; Dreger et al., 1957; Ma, 1999; Richardson et al., 1972). In a meta-analysis, Hembree (1990) reported a significant negative correlation between Math Anxiety and grades in mathematics in both high school (mean $r = -0.30$) and college (mean $r = -0.27$). Similarly, Math Anxiety was inversely related to scores in mathematics achievement tests in college students (mean $r = -0.31$) and fifth to twelfth graders (mean $r = -0.34$). A further statistical differentiation of this correlation revealed that the inverse relationship holds true for all subtests of mathematics achievement tests, i.e. computation, mathematical concepts, problem solving, abstract reasoning, and spatial ability. While previous studies have provided important insights into how Math Anxiety relates to performance in arithmetic and other school mathematics, they did not take other measures into account that might mediate the anxiety-performance link.

Math anxiety might also influence performance more directly, by overloading working memory (Ashcraft et al., 1998). Anxious people are likely to have intrusive thoughts about how badly they are doing, which may distract attention from the task or problem at hand and overload working memory resources. Many studies associate working memory deficits with general anxiety as a trait (Dowker et al., 2016). Affecting working memory would make a strong effect on arithmetic highly probable; many studies find working memory strongly associated with arithmetical performance, especially in tasks involving multi-digit arithmetic, with possible carrying required (Dowker et al., 2016). The load that Math Anxiety and associated ruminations place on working memory could thus be a plausible explanation for decrease in math performance. Ashcraft and Kirk (2001) found smaller working memory spans in those with high math anxiety than those with lower Math Anxiety, especially in tasks requiring calculation. In particular, high Math Anxiety made sufferers much slower and more prone to errors than others in tasks requiring mental addition along with keeping numbers in memory. Such tasks probably activate or increase state mathematics anxiety.

3.1.2.B Psychological

Poor mathematical attainment may also lead to mathematics anxiety, as a result of repeated experiences of failure. Math Anxiety negatively affects math efficacy (students’ belief in their ability to solve specific mathematics tasks), self-concept (students’ beliefs in their own abilities) (Ahmed et al., 2012; Sorvo et al., 2017). Yenilmez et al. (2007) describe several psychological characteristics associated with Math Anxiety. First, extreme individuals feel helplessness, often paired with panic when faced by math problems. Sometimes such students will express feelings that they have reached their limits in math and cannot possibly achieve any further understanding. Additionally, individuals with Math Anxiety often display paranoia, feeling as if they are isolated in their anxiety and that others are aware of their state of mind. This characteristic is especially relevant in classes where math anxious individuals feel that all their classmates understand the lessons, in contrast to their perceptions of being the only student failing to comprehend the material. Just as in depressed people, those suffering from Math Anxiety also exhibit passive attitudes toward math, and their perceived external locus of control; e.g. a math anxious student may feel that they lack a “math mind”, making further learning of math beyond their capacity. Lack of confidence also typifies such people.

They feel unable to improve at math computations, and reliance more on “rule memorization” rather than conceptualizing theories behind calculations they are being taught. Behaviorally, they tend to become jittery, sweat profusely, and exhibit hyperventilation. Others become despondent, showing apathy and accepting poor performance. In examinations, highly math anxious students may exhibit avoidance behavior by rushing through them (Beilock et al., 2005) or perseverate excessively on particular problems and so run out of time.

3.1.2.C Cognitive

Mathematics learning requires the student to master specialized concepts and procedures in many areas of study. The challenges of teaching and learning mathematics can be understood and overcome by analyzing cognitive processes. Children require good cognition to perform well in school, as in challenges in later life (Puerta Morales, 2015). Anxiety reaction causes falls in working memory, which underlies successful mathematical processing, as shown in many studies (Witt, 2012).

Attentive behaviour improves learning. Attention span is also an important determining factor for computational ability (Fuchs et al., 2006). Evidence suggests that test anxiety disturbs cognition, decreasing children's attention span and bringing negative, off task, thoughts. Certain mental processes like negative self-talk, play important roles in maintaining, if not causing, anxiety in children. (Lodge et al., 1995) Similar processes in anxious adults result in senses of unpredictability and threat, even danger (Greenberg et al., 1989). These may also be seen in children. In establishing Children's Cognitive Assessment Questionnaire (CCAQ), Zatz and Chassin (1983) observed children with high test anxiety to score higher on off task and negative thoughts. Conversely, they found higher rates of positive thought in children with low anxiety. But another study failed to find significant correlations (Prins et al., 1994). Restructuring self-beliefs might therefore offer a partial solution to Math Anxiety.

Math Anxiety disrupts cognitive processes (Suárez-Pellicioni et al., 2015). Anxiety alone can tax working memory so much that individuals with high aptitude for maths may perform poorly (Beilock et al., 2005). Past research suggests that anxiety depletes working memory resources by creating a dual-task situation (Ashcraft, 2002; Ashcraft et al., 2007). By overloading working memory, it may also influence performance more directly (Ashcraft et al., 1998). Anxious people experience intrusive thoughts suggesting they are doing badly, distracting their attention from the task at hand, and overloading working memory resources. Many studies have identified general anxiety as a trait associated with working memory deficits (Dowker et al., 2016).

For arithmetic, it is likely that, if anxiety affects working memory, it has a very strong effect on arithmetic. Many studies have found working memory strongly associated with performance on arithmetic, especially in multi-digit tasks, or those requiring carrying (Dowker et al., 2016). The load that Math Anxiety and associated thought processes place on working memory thus offers a plausible explanation for observed decreases in maths performance.

3.1.3 Mechanisms of Math Anxiety:

Different measures of Math Anxiety are more strongly correlated between themselves than Math Anxiety with test anxiety or general anxiety (Dowker et al., 2016). Math Anxiety is a good predictor of performance on mathematics tests even after controlling for test anxiety (Lukowski et al., 2016). In students with high Math Anxiety, anticipation of doing math induces greater visceral threat detection activity, and similarly increased activity in the dorsoposterior insula associated with experience of pain, than in those with low Math Anxiety (Lyons et al., 2012). This does not occur when those high in Math Anxiety anticipate doing a reading activity. Math Anxiety thus appears specific to mathematics; it does not imply that other academic domains such as reading will cause anxiety.

Two dimension of Math Anxiety have been identified: cognitive and affective (Ho et al., 2000). “Worry” is a cognitive dimension referring to concern for performance, and possible consequences of failure. “Emotionality”, part of the affective dimension, comprises nervousness, tension and their respective autonomic reactions in test situations (Liebert et al., 1967).

Sutter (2006) lists brain processes occurring during Math Anxiety. Anxiety primarily affects the limbic system with its four main component parts in the center of the brain above the brainstem: the thalamus, hypothalamus, amygdala and hippocampus (Sousa, 2006; Jensen, 1998, in Sutter, 2006). The limbic system is responsible for emotions, sleep, and the production of endocrine hormones etc. The thalamus receives data from the senses, which it filters for survival content, semiotically evaluating its significance by referring back to memories of past experiences (Sousa, 2006, in Sutter, 2006). Survival being a “high priority” means that its judgments are processed immediately, with possible increase in pituitary hormone levels like epinephrine etc. The thalamus and hypothalamus are deeply involved in anxiety reactions. So is the amygdala, which transmits emotional messages to long-term memory. Amygdala activity ensures that emotions felt while learning lessons are included whenever lessons are recalled. Recalling an experience is sufficient to re-awaken its emotions and physical manifestations (Sutter, 2006). Jensen explains how anxiety affects and inhibits normal brain processes required for learning (Sutter, 2006).

Math Anxiety disturbs visual and verbal working memory thus impacting math performance. This in turn affects emotional factors (Dreger et al., 1957). In general anxiety disorder and in anxiety prone individuals, the brain’s areas of emotion like the amygdala (Rauch et al., 2003) and insula, are activated (Etkin et al., 2007; Stein et al., 2007). Through hyperactivity of the right amygdala, Math Anxiety impairs prefrontal mechanisms regulating negative emotions. It also reduces posterior parietal cortex connectivity (Young et al., 2012). Two neural networks responsible for emotionality connected to Math Anxiety have been identified: the fear network centered around the amygdala (Young et al., 2012), and the pain network involving the insula (Lyons & Beilock, 2012b).

The last authors also showed that those with high Math Anxiety exhibit reduced activity in frontal and parietal areas that anticipate and carry out maths tasks compared to normal individuals (Lyons & Beilock, 2012a). Such individuals perform less well on maths tasks. During task performance they exhibit high activation of subcortical areas associated with motivation and assessment of risk and reward, rather than the parietal and other cortical areas responsible for doing arithmetic. We may therefore conclude that Math Anxiety alters brain activity patterns, including sub-region connectivity – all mediated by emotional regulation.

During math tasks, highly math-anxious children show greater coupling of the amygdala to cortical regions involved in processing and regulating negative emotions (Young et al., 2012), leading to deactivation of the ventromedial prefrontal cortex, compared to those with low Math Anxiety. In math-anxious individuals, neural activation in the left inferior frontal gyrus and the insula, typical of processes of place-value integration in multi-digit numbers failed (Pletzer et al., 2015). During number comparison tasks, these areas exert inhibitory control. Results therefore suggest that Math Anxiety inhibits emotional processing in brain areas not relevant to the task, rather than activating inhibitory control regions that would aid task performance (Pletzer et al., 2015). Math-anxious individuals thus have their attention diverted to their math-related emotions, rather than being able to focus on the task itself, something clearly bad for task performance. But neuroscientific strategies involved in emotion regulation still need deeper understanding (Gross et al., 2003).

Pletzer et al., (2010) found no relation between Math Anxiety and cortisol levels, indicating that other factors are causing the correlation. Mattarella-Micke et al., (2011) reported measurements of cortisol levels after math performance in high working memory students. Individuals with both higher working memory and math-anxiety showed anticorrelation between salivary cortisol concentration and math task performance, i.e. performance was compromised in those with high salivary cortisol. In contrast, in those with higher working memory and low math-anxiety, higher salivary cortisol concentrations seemed to improve performance. This seems to exemplify the relationship between Hans Selye’s distinction between ‘eustress’ that improves performance, and ‘distress’ that compromises it.

Núñez-Peña and Suárez-Pellicioni, (2014, 2015) studied a combination of event-related potential (ERP) and behavioral measures of numerical processing in those with high and low Math Anxiety (as measured on the MARS questionnaire). In a magnitude comparison test, those with high Math Anxiety had slower reaction times, and showed larger size and distance effects than those with low Math Anxiety. The ERP indicated that high Math Anxiety individuals had higher amplitude in frontal areas associated with numerical processing for both size and distance effects than those with low Math Anxiety. They also investigated two-digit addition in people with high and low Math Anxiety, by presenting them with both correct and incorrect answers, and requiring them to give right or wrong marks for each answer. Participants with high Math Anxiety were slower and less accurate than those with low Math Anxiety. In people with high Math Anxiety the ERP P2 component associated with giving attention to emotionally negative stimuli was larger than in those with low Math Anxiety. Such studies suggest that people with high Math Anxiety devote extra attentional resources to their worries, decreasing task performance (though correlational studies do not determine direction of causation.) A newly published study (Schillinger et al., 2018) found Math Anxiety inversely related to numerical and figural intelligence, but not to verbal intelligence. Math Anxiety seems specifically related to processing numerical and visual-spatial information.

3.1.4 Treating Math Anxiety

Three kinds of strategy have been implemented in attempts to reduce Math Anxiety. These include (a) curriculum strategies like courses on how to deal with Math Anxiety, self-paced learning, distance education, retesting, and single-sex classes, (b) instructional strategies, like technology, self-regulation techniques, manipulatives, and communication, and (c) non-instructional strategies, such as relaxation therapy and psychological treatment (Iossi, 2007).

Treatments of already-established Math Anxiety may involve maths interventions, or anxiety treatments such as systematic desensitization and cognitive behavior therapy. So far, no miracle cure seems to be in sight. However, recent research findings suggest that some methods appear promising. Attempts to correct negative beliefs were moderately successful (Hembree, 1990), but classroom interventions were ineffective.

Johns et al., (2008) and Jamieson et al., (2010) both report that informing people that arousal can improve performance improved test results compared to control conditions. Writing down worries and concerns about maths classes can alleviate some Math Anxiety (Maloney et al., 2012; Park et al., 2014). These researchers drew on findings that writing about traumatic and highly emotional events can lower ruminating behavior in those with clinical depression (Smyth, 1998). Writing seems to free working-memory involved in worrying, which can then be used in task performance. Ramirez and Beilock (2011) tested this proposition both in the lab and a real exam. Their two experiments both showed that writing about one's worries significantly improves academic performance compared to writing about untested exam material.

The possibility that cognitive tutoring might help reduce Math Anxiety has been investigated. Supekar et al. (2015) evaluated an 8-week one-to-one math-tutoring program, Fuchs' MathWise, (Fuchs et al., 2013), aiming to improve maths skills in 7 to 9 year old children with Math Anxiety. The children underwent three sessions of 40–50 min mathematics tutoring per week, with Math Anxiety assessed on the Scale for Early Mathematics Anxiety (Wu et al., 2012). They also underwent an fMRI scan before and after the intervention. During scanning, an addition task or a number-identification, control task, was performed. The study indicated that tutoring reduced Math Anxiety. It also reduced activity in circuits associated with the baso-lateral amygdala in those with high Math Anxiety, but not low Math Anxiety. The study reported correlations between decreases in amygdala activity and reductions in Math Anxiety; it proposed that similar to exposure-based anxiety reduction in other anxiety disorders, sustained exposure to mathematical stimuli can reduce Math Anxiety, possibly by reducing amygdala activity.

Another possible treatment, which is just beginning to be explored, employs non-invasive brain stimulation, a technique that modulates neural activity over broad areas of the cortex. Transcranial electrical stimulation applies mild electrical currents to the scalp and is painless. It can up regulate or down regulate neuronal activity underneath the cortex (Dowker et al., 2016).

3.2 Prāṇāyāma

Prāṇāyāma, the enlivenment, ayāma, of Prāṇā, is the fourth of the eight limbs of Yogā (Nagendra, 2005). It exerts a deeply settling influence on the mind (Burke et al., 2007), helping a person remain in the present. It is a standard preparation for meditation (Brown et al., 2009). Its effect on the tissues can be measured electrically at acupuncture points (B. Sharma et al., 2014), while its settling influence has been confirmed by observations of test anxiety reduction, improved test performance (Nemati, 2013), perceived stress reduction, and improvement in cardiovascular parameters (V. Sharma et al., 2013; Sivapriya et al., 2010), also reducing aggression (Deshpande et al., 2008). Breathing techniques such as Nāḍī Śodhana, Sudarshan Kriya, and Bhastrikā utilize rhythmic breathing to guide practitioners into states of deep relaxation, promoting self-awareness.

3.2.1 Prāṇāyāma for Anxiety Reduction

Yogā Prāṇāyāma is well known to improve conditions in both clinical and non-clinical situations. Evidence that Prāṇāyāma reduces stress levels and improves individual wellbeing is strong (Brown et al., 2009). Prāṇāyāma achieves profound results in anxiety reduction (Kirkwood et al., 2005).

Studies have shown that 3 months' regular practice of slow Prāṇāyāma decreases perceived stress and improves the cardiovascular functions: heart rate, systolic and diastolic blood pressure (Naik et al., 2013) as well as pulmonary functions (Gaur et al., 2015) in the healthy volunteers. A program incorporating Prāṇāyāma, Yogā, and relaxation training found that severity of depression was greatly reduced at one and three-month follow-ups (Bridges et al., 2017). Prāṇāyāma can reduce state anxiety in people suffering from epilepsy (Jayachandran et al., 2017) and neurosis (Kumar et al., 2017). In a comparison of techniques positively impacting mood and energy (physical *and* mental), Prāṇāyāma produced a significantly greater positive impact than visualization and relaxation techniques (Wood, 1993). Deep breathing exercises are found to reduce PTSD-like symptoms, normalizing high cortisol levels (Kim et al., 2013). Yogā Prāṇāyāma exercises have been found effective in reducing student anxiety at many institutions (Varambally et al., 2016). They also reduce aggression (Deshpande et al. 2008).

Regarding breathing: Anxiety and stress activate the sympathetic nervous system (Pohjavaara et al., 2003), but several studies have found that, during respiration, inhalation inhibits sympathetic nervous system activity (Jerath et al., 2015). In contrast, irregular breathing like Cheynes-Stokes respiration involving shallow and/or deep breathing and periods of apnea, leads to sympathetic nervous system activation (Leung et al., 2006). But slow deep breathing leads to near-complete sympathoinhibition (Seals et al., 1990), inversely proportional to lung volume: high lung volumes maximize inhibition (St Croix et al., 1999), which occurs during the second half of inspiration. For example, during slow deep breathing, inhibition occurs from mid-inspiration to early-mid expiration (Seals et al., 1990). However, subjects performing deep breathing with higher starting lung volume experience sympathoinhibition earlier during inhalation. Findings suggest that depth of breathing, breathing pattern, and starting lung volume all influence degree of sympathoinhibition, with slower, deeper breathing creating highest levels of inhibition (Seals et al., 1990).

These results are prime examples of respiration directly modulating the ANS by inhibiting the sympathetic nervous system as well as respiration affecting membrane potential. Generally, Prāṇāyāma practice increases thoracic volume. Resulting inhibitory signals may induce changes in the ANS leading to increased parasympathetic dominance (Nivethitha et al., 2016). The pathophysiology of panic disorder may involve decreased GABA levels (Goddard et al., 2001), but after a single Yogā session, consisting of postures, Prāṇāyāma, meditation, and chanting, participants showed 27% increase in GABA levels (Streeter et al., 2010).

In general, Prāṇāyāma has a deeply settling influence on the mind (Burke et al., 2007) helps a person to be in the present, i.e. ‘mindful’, and enlivens Prāṇā in the tissues, as measured electrically at acupuncture points (Sharma et al., 2014). Measures like anxiety reduction, improved test performance (Nemati, 2013), perceived stress reduction, and improvement in cardiovascular parameters confirm its settling influence (Bhavanani, Raj, Ramanathan, & Trakroo, 2016). The many studies published on various

Prāṇāyāma programs mean that its effects are well established. It can therefore be used as a standard to compare with other methods being assessed. Prāṇāyāma represents a recognized way of managing mild levels of stress, improving emotional balance and regulation, and decreasing aggression. Practice usually involves focusing on the breathing process, bringing the mind to the present moment. Improving that ability will be of value to taking tests.

3.2.2 Prāṇāyāma and Cognitive Skills

An ancient Samskrit text, Śivaswarodaya, describes how breathing through each nostril affects cognitive activities and emotions of an individual. Uninostril breathing is found to be useful in improving memory, attention, reaction time, and other aspects of cognition (Ghiya, 2017). Uninostril breathing (left nostril breathing or right nostril breathing) and alternate nostril breathing (Nāḍī Śodhana Prāṇāyāma), bring positive changes in cognitive tasks (Naveen et al., 1997). These techniques help sharpen the critical faculty and creativity, and bring balance between left and right hemispheres of the brain (Nagendra, 2005). Both fast (Kapālabhāti, Bhastrikā) and slow (Nāḍī Śodhana) Prāṇāyāmas reduce stress levels, as does Prāṇāva (AUM) chanting, and improve cognitive skills, particularly working memory (V. K. Sharma et al., 2014).

A study by Rajesh et al. (2014) showed that Bhrāmarī Prāṇāyāma shortens reaction time and improves cognitive flexibility on the Stop signal task in healthy volunteers. The first indicates improved sensory-motor performance and enhanced CNS processing ability. This may be due to greater arousal, faster rate of information processing, improved concentration and / or ability to ignore extraneous stimuli (Bhavanani et al., 2003).

Prāṇāyāma can improve cognitive brain functions impaired by disease (Sengupta, 2012). Prāṇāyāma exercises have been found effective on attention/orientation, language, memory, verbal fluency and visuospatial domains in individuals with type 2 diabetes (Nagothu et al., 2017). Total errors on a six-letter cancellation task decreased after practicing Kapālabhāti but not for a group practicing breath-awareness. The same study found improved mean score after Kapālabhāti compared to breath awareness (Telles et al., 2008). The study proved that Prāṇāyāma improves selective and sustained attention as well as ability to shift attention. Another study by Telles et al. (2017) found decreased time to perform a digit vigilance task (better sustained attention) after practice of Nāḍī Śodhana, combined with decreases in systolic and mean blood pressure. Telles et al. (2012) found that right nostril breathing facilitates left hemisphere activity significantly reducing latency of the P300 evoked potential. This neuroelectric event arises from interaction between the frontal lobe and hippocampal and temporoparietal functions (Telles et al., 2012). It thus improves interhemispheric activity and helps enhance: (a) cognitive task performances and (b) integrated EEG amplitudes.

Functional near-infrared spectroscopy (fNIRS) is a noninvasive optical method of measuring real time change in oxygenated hemoglobin (oxyHb), deoxygenated hemoglobin, and total hemoglobin (i.e. blood volume) in different brain regions including bilateral prefrontal cortices (PFCs). A study using fNIRS assessed effects of Kapālabhāti on PFC blood flow changes in 18 healthy persons and 18 with schizophrenia. Significant increase in bilateral prefrontal oxyHb were observed in the healthy subjects (Bhargav et al., 2014), suggesting that Yogā breathing improves brain hemodynamics. All this suggests that Prāṇāyāma constitutes a cognitively stimulating activity promoting brain health.

3.2.3 Prāṇāyāma for Children/Adolescents

Prāṇāyāma may play a significant role in preserving and "restoring" both physiological and cognitive functional reserve in school children. In a comprehensive review proposing the implementation of Yogā and meditation in education, J. Davidson et al. (2012) suggested that benefits of Yogā observed in adults are also relevant for children and adolescents in school settings. Research on Yogā interventions in schools is in its early stages, but initial results are promising. For example, Serwacki and Cook-Cottone (2012) reviewed 12 preliminary studies of Yogā in schools concluding that Yogā interventions exert positive effects on factors such as emotional balance, attentional control, cognitive efficiency, anxiety, negative thought patterns, emotional and physical arousal, reactivity, and negative behavior. Other research suggests positive effects of school-based Yogā programs on aspects of mental health: concentration, attention, anxiety, stress, mood, resilience, emotional arousal, self-esteem, and coping frequency (Butzer et al., 2015). Yogā in school helps stress management, as it lowers children's cortisol levels (Butzer et al., 2015). Prāṇāyāma decreases stress among students caused by exam results the previous year, or attending morning classes (Joseph et al., 2017).

Clinically, children gain significant results from Yogā practice (Galantino, Galbavy, & Quinn, 2008). The review of 24 studies of Yogā for youth (Galantino et al., 2008) concluded that Yogā practices improve neuromuscular, cardiopulmonary and musculoskeletal areas. The review also found Yogā effective in helping children suffering from ADHD, asthma, fear and anxiety; and that cognitive skills improved.

3.3 VEDIC MATHS

3.3.1 Origin and Uniqueness of Vedic Maths

Vedic Maths is the legacy of Swami Bharati Krsna Tirthaji Maharaja, Jagadguru Śaṅkarācārya of Purī (1884-1960), who took the vows of sanyāsa after working as a teacher of high school mathematics. The book of that name, (Bharati Krsna Tirthaji Maharaja, 1992) is a posthumous summary of lectures on the subject given in his lifetime. These covered basic high school arithmetic and algebra, and proved immensely popular. As a result methods that it presents have been expanded to include geometry, coordinate geometry, trigonometry and differential and integral calculus (Nicholas et al., 2010; Williams, 2009),. The system presents specific conceptual patterns to aid problem solving in all fields of high-school mathematics.

In contrast to ordinary maths teaching at such levels, the system has intuitive and holistic natures (Price, 1997; Weinless, 2011). One book (Glover, 2005a) suggests that Vedic Maths can be used in high school mathematics teaching as a supportive aid alongside conventional mathematics teaching approaches. Teachers experienced in its use report that students enjoy applying its methods, and may gain or regain liking for mathematics. More widely, its methods have been applied to design of computer chips, with decreased computer chip areas shortening times taken by algorithms used in multiplier circuits. (Anjana et al., 2015).

3.3.2 Criticism on Vedic Maths

Critics of Vedic Maths have raised questions concerning how Vedic it may be. (Chandra Hari, 1999) Some who criticise its name acknowledge that it may reduce math phobia (Dani, 1993). Though the name may have provoked controversy, as discussed in Chapter 2, the style is definitely in the form of Vedic aphorisms using words from Vedic

Sanskrit (see Table 1) (Kansara, 2000). All words used in Vedic Maths Sūtras are taken from various Sūtra sections of Sthapatyaveda, the Upaveda of Atharvaveda, such as the Śulba Sūtra and Śrota Sūtra texts used in building altars.

Regarding direct criticism of the Vedic Maths book: Chandra Hari has observed that the Sūtras of Vedic Maths are not to be found in Atharvaveda, concluding that the epithet 'Vedic' is inappropriate (Chandra Hari, 1999). They are merely descriptions of arithmetical or algebraic processes, and don't fall under modern theorems or axioms. They rather suggest different calculation processes for use in different contexts. They describe short cuts rather than algorithms (Chandra Hari, 1999). Camouflaged algebraic patterns are presented, and explained using Sanskrit terms in attempts to connect them with Veda. Some have suggested that, instead of 'Vedic Maths', the name pattern mathematics would be more suitable (Rao, 2016). Here one should note that such critics accepting these as patterns, but do not agree that the Sūtras are like those of the Vedic era. They suggest including Bhāskarācārya′s Līlāvati in modern education supplemented by the Vedic Maths shortcuts manage 'maths phobia' (Rao, 2016).

Comments: Patterns have their own place. The rules in the Vedic Maths aphorisms were constructed by following clues in Sthapatyaveda. These two series of 29 formulae suggest various patterns for use in different contexts to simplify calculations by eliminating numbers of steps.

A paper by Dani (2012) states that, although Vedic Maths has controversial origins, it is a wonderful discovery, which helps remove high-school students' fear of maths. Dani says that sometimes practice is required to learn the use of specific patterns. The element of surprise given by these methods can be enhanced by well-chosen examples, to which each applies, he says, but they do not constitute general statements that can be used as axioms from which to derive proofs. In this, he misunderstands the system, attempting to put it into the straight jacket of modern mathematics. In reality, each Sūtra constitutes a ‘pattern of creative thinking’, which can be used like a key to unlock specific problems and solve them. However, the same Sūtra pattern-key may be used to unlock problems in different areas of mathematics. It was the genius of Ken Williams to see this, and to apply the Sūtras to many more areas of high-school mathematics than those originally envisaged (Nicholas et al., 2010). There is thus a valid sense in which the Sūtras are ‘general’.

Kandasamy and Smarandache (2006) conducted an extensive literature review on Vedic Maths. Their research into student appreciation using fuzzy logic concluded that it attempts to forcefully introduce into schools the Śaṅkarācārya′s approach to mathematics under the name ‘Vedic’. Instead it should have been called Vegagaṇita, i.e. ‘fast mathematics’.

To summarize: its critics suggest that boasting about Vedic Maths is not scientific. But accepting the calculational patterns introduced by Swami Bharathi Krishna Tirthi as calculational procedures can accelerate problem solving in a relaxed and artistic, almost recreational way. They can make learning mathematics fun and remove maths phobia.

3.3.3 Research Supporting Vedic Maths

Use of Vedic Maths methods can improve mathematical achievement of grade 5 students (Kaur at al., 2012). In one study, a 16-bit Vedic multiplier, 16-bit multiply accumulate unit and 16-bit arithmetic module, were designed using ‘Vertically & Crosswise’ (Ūrdhva Tiryagbhyāmaṁ) Sūtra, built and compared (Singh, 2010). The computational delay for the MAC unit and arithmetic module were found to be 11.151ns and 15.749ns, demonstrating that the Sūtra presents a more efficient multiplication procedure than those in current use.

Reddy and Reddy (2014) proposed a pipelined multiplier architecture for signed q-format multiplications using Vedic Maths' Ūrdhva Tiryagbhyāmaṁ Sūtra approach. This multiplier architecture substantially reduces the multiplier chip's occupied area.

At an international conference on Vedic Maths, the author has shown how identifying algebraic patterns associated with Vedic Maths in quadratic denominators can simplify their integration. Details are given in Appendix 1. The same appendix discusses problem-solving methods emerging from the author's observations of Vedic Maths, which incorporate short cuts permitting fast mental calculations. The new format uses derivative and discriminant of quadratic formulae allowing students to quickly select correct options which reduce eight to ten-line calculations to four, greatly increasing students' speed of solving otherwise laborious problems. These particular patterns need to be presented in the right way, at the right time, otherwise they do not appear to simplify what has to be done to any great extent. They are particularly appreciated when they are explicitly seen to reduce the labour involved in normal procedures. The methods may also be used to shorten algorithms in calculator and other applications requiring answers to quadratic integrals.

Experience of teaching students using these methods to recognize algebraic patterns shows that they help stimulate student interest and enjoyment when learning Integral Calculus. They also increase student confidence in working examples, correspondingly improving performance on examples, tests etc. Thus, the new quadratic integral formats, listed in terms of derivative and discriminant, can help both students and teachers improve their speed of simplification. They have proved efficient, short cut methods of enhancing student confidence and competence.

Vedic Maths methods, being a powerful teaching aid, prove popular with students (Syed Ismail et al., 2010). They develop abilities to visualize patterns of calculation for solving problems, enabling students to visualize how and why each method works. This increases understanding, and confidence when faced by exam questions. Those who are good at applying basics of high school mathematics and making speedy calculations naturally score well in entrance examinations where calculators are not allowed. On this basis we hypothesized that using Vedic Maths would improve cognitive skills, reduce Math Anxiety in 12th grade students, and improve those all-important scores on 12th grade mathematics exams. Any strategy reducing Math Anxiety and improving cognitive skills and subject results will help expand students' career choices.

3.3.4 Recent Development in Vedic Maths

According to Maharshi Mahesh Yogi, "Vedic Maths is that one field of knowledge which fulfills the purpose of education by developing the total creative genius of the individual, giving him or her the ability to be always spontaneously right, and automatically precise, so that his or her action, supported by Natural Law, is effortlessly fulfilling." Vedic Maths certainly brings the realization that math can be a fascinating and interesting subject, so that may be true.

In recent years, an Institute for the Advancement of Vedic Maths (IAVM) has been founded to promote the subject internationally. Two international conferences have been held, one in Kolkata in December 2016 and one in Delhi in December 2017. The first was sponsored by the Indian Council for Cultural Research (ICCR) a government body. In addition, an International Conference on Ancient Indian Languages in

Ahmedabad in September 2017 sponsored by Indira Gandhi National Open University included a section on Vedic Maths. A third conference is scheduled for August 2018. In addition, IAVM has hosted three online Vedic Maths conferences, with a fourth conference scheduled in August 2018.

3.4 SHORT SUMMARY AND CONCLUSION

Math Anxiety is statistically correlated with working memory and math performance. It brings high test anxiety, negative attitudes toward math teachers, low self-confidence. It develops at primary level and magnifies to university level course and disrupts cognitive skills. Math Anxiety is associated with slower solution rates for large arithmetic problems. Individuals with Math Anxiety often display paranoia and those with high test anxiety score higher on off task and negative thoughts. Two neural networks Amygdala and Insula are identified with connection to Math Anxiety. Student affected by Math Anxiety exhibit reduced activity in frontal and parietal areas. Individuals with both higher working memory and math-anxiety show anti-correlation between salivary cortisol concentration and math task performance.

To treat Math Anxiety, Vedic Maths method may be used, which suggests pattern-based approach to manage arithmetical and algebraic calculations. The system has intuitive and holistic natures and can be used as a supportive teaching aid to manage math phobia. It brings the idea that math can be a fascinating and interesting subject. Critics say that camouflaged algebraic patterns are explained using Sanskrit terms connecting them with Veda. Research shows that several terms used in sutras are taken from Śulba Sūtra and written in Vedic style. But patterns introduced by Swami Bharathi Krishna Tirthji can accelerate problem solving, in recreational ways, and can make learning mathematics fun and remove maths phobia.

Pranayama may be the alternative method to bring overall balance in personality, which may address the issues with negative emotions, aggression, stress reduction and cortisol levels. It helps a person to be in the present, i.e. ‘mindfulness’ and improve cognitive skills. Bhrāmarī Prāṇāyāma shortens reaction time and improves cognitive flexibility. Both Kapālabhāti, Bhastrikā Prāṇāyāmas reduce stress levels, and improve working memory. Nāḍī Śuddhi leads to better sustained attention. Right nostril breathing facilitates left hemisphere activity and helps enhance: (a) cognitive task performances and (b) integrated EEG amplitudes. Yoga breathing improves brain hemodynamics. Thus, Yoga Pranayama in schools exerts positive effects on cognitive efficiency, emotional balance, anxiety, negative thought patterns and behaviour.